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Lecture “Measure and Integration” (winter term 2010/11)

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3. Borel- and Lebesgue-measurable functions
   A) Definition and relation of both
   B) Lusin's theorem
   C) Carathéodory functions and the Scorza-Dragoni theorem
4. Approximation of measurable functions by step functions

V. The measure integral

1. Construction of the measure integral
2. Properties of the measure integral
3. Convergence theorems
   A) The Monotone Convergence theorem
   B) Fatou's lemma
   C) The Dominated Convergence theorem
4. Riemann and Lebesgue integral
   A) Comparison of both approaches
   B) The Riemann integral in the Lebesgue theory
5. Further properties of the measure integral (survey)
   A) Indefinite integral
   B) Lipschitz functions
   C) Parameter integrals
6. The $L^p(\Omega, \mu)$-spaces
   A) The L^p-norms
   B) The chain of L^p-spaces
   C) L^p-convergence
   D) The mean value theorem
   E) Baire classes

VI. Signed measures

1. Definition and basic properties
2. The Hahn decomposition of Ω
3. The Jordan decomposition of ν
4. Absolute continuity and the Radon-Nikodym theorem
5. Further decompositions of signed measures
6. Banach spaces of signed contents and measures
7. Signed measures as linear, continuous functionals
   A) Integration with respect to signed set functions
   B) The space $(\, C^0(\Omega)\,)^*$
   C) The space $(\, L^\infty(\Omega, \mathcal{B}^k_\Omega, \lambda^k)\,)^*$

VII. Applications of measure theory in the context of optimization

1. Identification of generalized derivatives as measures
   A) Schwartz distributions
   B) The space $BV(\Omega)$
   C) Second derivatives of convex functions
2. Measures as Lagrange multipliers related to state constraints
   A) State-constrained optimization problems in Sobolev spaces
   B) The subdifferential of convex functions
   C) The subdifferential for locally Lipschitz functions and the multiplier rule
3. Bochner integration
   A) Motivation and basic ideas
   B) Selected spaces of Bochner integrable functions
   C) Imbedding theorems
   D) The principal theorem of the calculus in the Bochner version
4. Young measures

VIII. A list of important topics not treated within this course