Dr. Marcus Wagner
Brandenburg University of Technology, Cottbus; Department of Mathematics, Chair for Optimization
Course “Calculus of Variations” (winter term 2003/04)

Course “Calculus of Variations” (winter term 2003/04) — Contents

VII. Constrained variational problems

1. A general multiplier rule
2. Application of the multiplier rule to (P_3)
3. Some special cases
   A) Basic problem (P_2) with incomplete boundary conditions
   B) Bolza problem
   C) Isoperimetric problem

VIII. Existence theorems

1. The generalized Weierstrass theorem
2. Convexity and the existence theorem for scalar problems
3. Quasiconvexity and the existence theorem for vectorial problems

IX. Examples for direct methods in calculus of variations

1. Variational problems with energy functionals
2. Equivalence between self-adjoint boundary value problems and variational problems (Q)
3. Equivalence between elliptical boundary value problems and variational problems (Q)
4. Solution of (Q) by the Ritz method
5. Solution of (Q) by finite element methods (FEM)
   A) Subspaces of continuous, piecewise linear functions
   B) Convergence of FEM with Courant triangles
   C) Remarks about the numerical realization of FEM

X. Regularity theorems

XI. Outlook

Seminar: List of topics

1. Mathematical modelling in the calculus of variations
2. Proof techniques in calculus of variations: the fundamental lemma and its generalizations
3. First-order necessary conditions for the basic problem
4. First-order necessary conditions for the basic problem (II)
5. Necessary condition for strong local minimizers
6. Canonical formalism
7. The multiplier rule
8. Second-order necessary and sufficient conditions for the basic problem
9. Second-order necessary and sufficient conditions for the basic problem (II)
10. Solution of BVP's by finite element methods