

Lecture “Optimization with Lipschitz Functions” (summer term 2008)

Contents

I. Lipschitz functions on \mathbb{R}^m

1. Subject of the lecture
2. Examples of Lipschitz functions
 - A) C^1 -functions
 - B) Pointwise maximum of Lipschitz functions
 - C) Distance function related to a closed set $C \subseteq \mathbb{R}^m$
 - D) The k th eigenvalue of a symmetric matrix
3. $\text{Lip}(\Omega, \mathbb{R})$ as normed vector space
4. Local Lipschitz continuity
 - A) Definition
 - B) Example: C^1 -functions
 - C) Example: Generalized convexity notions from the Calculus of Variations

II. Differentiability of Lipschitz functions

1. Measurable sets and measurable functions
 - A) Null sets
 - B) The system of the Lebesgue sets
 - C) Measurable functions
 - D) Approximation of measurable functions by semicontinuous ones
2. Selected topics from the Lebesgue integration theory
3. Differentiability almost everywhere of Lipschitz functions on \mathbb{R}
4. Differentiability almost everywhere of Lipschitz functions on \mathbb{R}^m
5. Notes and remarks
 - A) Local versions of the theorems from 3. and 4.
 - B) Weak derivatives
 - C) Lipschitz functionals on infinite-dimensional Banach spaces

III. The Clarke calculus

1. Basic ideas and definitions
2. Explicit computation of Clarke subdifferentials
 - A) C^1 -functions
 - B) Convex functions; maximum function
 - C) The Euclidean distance function $\text{Dist}(x, C)$
3. Calculation rules for the Clarke subdifferential
4. Extension to infinite-dimensional Banach spaces

IV. Nonsmooth optimization problems

1. Necessary optimality conditions
 - A) Unconstrained minimization
 - B) Problems with an inclusion constraint
 - C) Problems with equality, inequality and inclusion constraints
2. The gradient method
3. The subgradient method
 - A) The algorithm
 - B) A convergence theorem in the case of a convex objective
 - C) Remarks and examples concerning the implementation