

## Lecture “Convex Analysis” (summer term 2004) — Contents

### I. Introduction

1. The subject; historical remarks
2. Basic definitions and examples
3. Minkowski arithmetic
4. Recapitulation: Affine structures in the space  $X = \mathbb{R}^n$

### II. Convex sets (in finite-dimensional spaces)

1. Algebraic properties; the convex hull
2. Special case of convex cones
3. Topological properties of convex sets
4. Support and separation
5. Dual description of convex sets
  - A) The polar
  - B) The dual cone
  - C) Separation theorems for convex cones
  - D) Application to convex optimization problems with inequality constraints
6. Structure of the (relative) boundary of convex sets
  - A) Extremal points and faces
  - B) Facial structure of closed convex sets
  - C) Representation of convex sets by its extremal points

### III. Polytopes and combinatorial problems of convex analysis

1. Graphs and polytopes
2. Planar graphs, 3-dimensional polytopes and the Euler formula
3. The Euler-Poincaré theorem

### IV. Convex functions (on finite-dimensional spaces)

1. Algebraic properties; convex envelope
2. Convexity tests for differentiable functions
3. Properties of convex functions with relevance for optimization
4. Analytical properties of convex functions
  - A) Continuity and local Lipschitz continuity
  - B) Directional derivatives and the total differential
  - C) Subdifferential calculus
5. Dual description of convex functions

### V. Convex bodies (in finite-dimensional spaces)

1. The support function
  - A) Definition and properties
  - B) Bijective correspondence between (nonempty) compact, convex sets and support functions
  - C) Description of geometrical characteristics by means of the support function
2. The Hausdorff distance; approximation theorems
3. The Brunn-Minkowski inequality

### VI. Fundamental theorems of convex analysis in infinite-dimensional spaces