

Lecture “Variational and Optimal Control Problems in Image Processing” (summer term 2007) — Contents

I. Images

1. Human vision
2. Computer vision
3. Mathematical models of images
4. Remarks about the function spaces used in image processing
 - A) Sobolev spaces
 - B) The space $BV(\Omega)$ of functions of bounded variation

II. Basic problems of image processing in variational formulation

1. Image restoration / image smoothing
 - A) Problem formulation
 - B) Classification of regularization terms
2. Image restoration & edge detection
3. Image deblurring
4. Image inpainting
5. Image registration / image matching
6. Optical flow
 - A) Problem formulation
 - B) Applications
 - C) Variational formulation
 - D) Classification of regularization terms
 - E) Edge detection within the optical flow
7. Shape from Shading
 - A) The Horn equation
 - B) Variational formulation involving higher partial derivatives of X

III. Multidimensional calculus of variations: image restoration as model problem

1. Existence of global minimizers
 - A) Image restoration with quadratic regularization
 - B) The generalized Weierstrass theorem
 - C) Image restoration with convex regularization for greyscale images
 - D) Quasiconvex functions
 - E) Image restoration with quasiconvex or convex regularization for color images
 - F) Image restoration with TV regularization for greyscale images
2. Tichonov regularization
 - A) Ill-posed operator equations and variational problems
 - B) The regularization parameter μ in the image restoration problem

3. Remarks about numerical methods for image restoration

A) Quadratic regularization in the Hilbert space $W^{1,2}(\Omega)$

B) Convex, nonquadratic regularization in the reflexive Banach space $W^{1,p}(\Omega)$

C) TV regularization and duality

IV. Basic problems of image processing in optimal control formulation

1. Image restoration & edge detection

2. Optical flow & edge detection

3. Shape from Shading

A) Optimal control reformulation

B) The gradient restriction for the solutions of the Horn equation

V. Dieudonné-Rashevsky type problems: Shape from Shading as model problem

1. Existence of global minimizers

2. The Pontryagin maximum principle