

Lecture “Fourier Analysis” (summer term 2003) — Contents

I. Introduction: expansions into series within function spaces

1. What are Fourier series, and why they should be studied?
2. Expansion within spaces of individual functions / power series
3. Expansion within spaces of equivalence classes of functions / trigonometric series

II. L^2 -theory of Fourier series

1. Survey: The Lebesgue integral
2. Spaces of complex-valued functions on $[-\pi, \pi]$
3. $L^2[-\pi, \pi]$ as Hilbert space
4. Basic concepts of Fourier series
 - A) Properties of the complex exponential function
 - B) Fourier coefficients
 - C) Completeness of \mathcal{G}
 - D) Fundamental theorems about Fourier series in $L^2[-\pi, \pi]$
 - E) Dense subsets in $L^2[-\pi, \pi]$
5. The inverse problem
6. Remarks about Fourier series of L^1 -functions

III. Pointwise convergence of Fourier series

1. L^2 -norm convergence and pointwise convergence
2. Basic facts about pointwise convergence of Fourier series
3. Dirichlet kernel and Dirichlet integral
4. Pointwise convergence for different classes of real-valued functions
 - A) Monotone functions
 - B) Functions of bounded variation
 - C) Lipschitz functions and C^k -functions
5. Summary: Everywhere pointwise representation of C^k -functions
6. The rate of convergence depending on the existence of derivatives of f
7. The Gibbs phenomenon

IV. Cesàro limitation of Fourier series

1. What does Cesàro limitation mean?
2. Fejér kernels
3. The Fejér convergence theorem
4. Uniform convergence of Fourier series
5. The Weierstrass approximation theorem
6. Remarks: Cesàro partial sums and Fourier series of L^1 -functions

V. The Fourier transform

1. Motivation and basic idea
2. L^p -spaces of functions with infinite domains
3. The convolution: “multiplicative” operation with $L^1(-\infty, \infty)$ -functions
4. The Fourier transform and its inverse
 - A) Definition
 - B) Properties of the Fourier transform
 - C) Examples
5. The Fourier inversion theorem
6. The Riemann-Lebesgue theorem

VI. A list of important topics not treated within this course