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Lecture “Set-Valued Maps” (winter term 2011/12)

Lecture “Set-Valued Maps” (winter term 2011/12) — Contents

I. What are set-valued maps, and why they should be studied?

1. Natural extensions of the concept of a function
2. Typical situations where set-valued maps occur:
   - A) Geometry
   - B) Inverse function
   - C) Subgradients
   - D) Optimization
   - E) Differential inclusions

II. Sequences of sets

1. Convergence in the sense of measure resp. probability theory
2. Convergence in the sense of Painleve / Kuratowski
3. The K-limit and convergence with respect to the Hausdorff distance
4. Notes and remarks

III. Application: Fractal image compression

1. Motivation and basic ideas
2. Contractive mappings on the metric space $[\mathcal{A}, \mathcal{H}]$
3. Binary images and fractal image compression by means of global IFS
4. Greyscale images and fractal image compression by means of local IFS

IV. Semicontinuous and continuous set-valued maps

1. Definitions. Inverse images
2. Semicontinuity and continuity
3. Continuous selections for semicontinuous set-valued maps
4. Lipschitz continuity (survey)
5. Applications: differential inclusions, the Graves theorem

V. Measurable set-valued maps

1. Measurable functions and products of σ-algebras
2. Measurable set-valued maps
3. Measurable selections for measurable set-valued maps
4. The Filippov theorem

VI. Application: existence theorems in optimal control

1. Differentiability and Lipschitz functions
2. The existence theorem for the problem with Mayer objective
3. The existence theorem for the problem with Bolza objective
4. Examples
   - A) Soft landing of a spacecraft
   - B) Image smoothing with simultaneous edge detection
   - C) A counterexample: Shape from Shading

VII. A list of important topics not treated within this course