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Lecture “Analysis III: Integration and Ordinary Differential Equations” (winter term 2006/07)

Lecture “Analysis III: Integration and Ordinary Differential Equations” (winter term 2006/07) — Contents

A) Applications of the Riemann integration theory in geometry and physics

I. Line and surface integrals of the first type

1. Curves in \mathbb{R}^n
2. Rectifiable Jordan curves
3. Line integrals of the first type
4. Regular surfaces in \mathbb{R}^3
5. Content of regular surfaces and surface integrals of the first type
6. Notes and remarks

II. Line and surface integrals of the second type

1. Vector fields in the space \mathbb{R}^3
2. Line and surface integrals of the second type
3. Potential fields
4. Solenoidal fields
5. Helmholtz-Weyl decomposition of three-dimensional vector fields
6. Extension to higher dimensions $n \neq 3$

III. Integral theorems

1. Preliminary remarks concerning methodology
2. The Gauss-Ostrogradsky theorem in \mathbb{R}^2
3. The Stokes theorem in \mathbb{R}^3
4. The Gauss divergence theorem in \mathbb{R}^3
5. Applications of the Gauss’ and Stokes’ theorems

B) Two extensions of the Riemann integral notion

IV. The Riemann-Stieltjes integral

1. Introduction. Functions of bounded variation
2. Definition and basic properties of the Riemann-Stieltjes integral
3. Existence, computation, geometrical interpretation
4. Applications

V. The Lebesgue integral

1. Basic ideas
2. Introduction of the Lebesgue integral via monotone approximation by simple functions
3. Properties of the Lebesgue integral and the function set $L(\Omega)$

4. The convergence theorems of the Lebesgue theory
 - A) B. Levi's monotone convergence theorem
 - B) Lebesgue's majorized convergence theorem
 - C) Fatou's lemma
5. The Riemann integral in the Lebesgue theory
6. Survey of further assertions within the Lebesgue theory
 - A) The indefinite integral
 - B) Lipschitz functions
 - C) Integrals depending on parameters
 - D) L^p -spaces and L^p -norms
 - E) Multiple Lebesgue integrals
7. Some applications of the Lebesgue integral

C) Ordinary Differential Equations

VI. Basic concepts

1. Classification of differential equations
2. Solution concepts for ODE's
3. Geometrical visualization of the meaning of an ODE
 - A) The directional field
 - B) The state space

VII. Elementary solution procedures for first-order ODE's

1. Direct quadrature
2. First-order linear equation
 - A) Homogeneous linear equation
 - B) Inhomogeneous linear equation
3. Bernoulli equation
4. Riccati equation
5. Exact differential equation
6. Separated differential equation
7. Remarks about the numerical solution of first-order ODE's
 - A) Euler-Cauchy's polygonal method
 - B) Quadrature formulas and the related one-step procedures

VIII. Fundamental theorems about first-order ODE's

1. The Banach space of the continuous functions $C^0[a, b]$
2. The Peano existence theorem
3. The Picard-Lindelöf existence and uniqueness theorem
4. Dependence on parameters
5. Reformulation of the fundamental theorems for systems of differential equations and n th order ODE's

IX. Linear differential equations

1. Global existence of solutions
2. Linear equations of n th order with variable coefficients
 - A) Homogeneous equation
 - B) Inhomogeneous equation
3. Linear equations of n th order with constant coefficients
 - A) Homogeneous equation
 - B) Inhomogeneous equation
4. Outlook on systems of first-order linear equations
 - A) Systems with variable coefficients
 - B) The matrix exponential function
 - C) Systems with constant coefficients

X. A list of important topics not treated within this course